Lesson 14. The Points-After-Touchdown Problem

1 The problem

- In an NFL football game, after scoring a touchdown, a team is given the option to try for:
  - a **1-point conversion**: 1 extra point by a field goal from the 15-yard line, or
  - a **2-point conversion**: 2 extra points by advancing the ball into the end zone from the 2-yard line
- Whether to "go for 2" is a classic debate – a few discussions on the topic:
  - https://fivethirtyeight.com/features/more-nfl-teams-are-going-for-two-just-as-they-should-be/
  - https://www.espn.com/nfl/story/_/id/28100383/going-2-8-points-why-nfl-teams-keep-doing-why-analytics-backs-up
- Adding to the debate: in 2015, 1-point attempts were moved from the 2-yard line to the 15-yard line
- Conversion success rates from the 2014-2017 regular seasons (from http://www.pro-football-reference.com/):

	2014	2015	2016	2017
1-point conversion success rate	0.993	0.942	0.936	0.940
2-point conversion success rate	0.483	0.479	0.486	0.451

- Based on the current score and time remaining, should a team "go for 1" or "go for 2" in order to maximize the probability that it wins the game?
- How does the 2015 rule change affect a team's optimal conversion strategy?
- Let's try to answer these questions by modeling this problem as a stochastic dynamic program
- We will be roughly following this paper:

  H. Sackrowitz (2000). Refining the point(s)-after-touchdown decision. *Chance* **13**(3): 29-34.

2 Data

- Two teams: A and B
  - Assume that we (the decision-makers) are Team A
- Suppose we have the following data:

$$\begin{aligned}
T &= \text{total number of possessions} \\
p_{1n} &= \Pr\{\text{1-pt. conv. successful for Team } n \mid \text{1-pt. conv. attempted by Team } n\} && \text{for } n = \text{A, B} \\
p_{2n} &= \Pr\{\text{2-pt. conv. successful for Team } n \mid \text{2-pt. conv. attempted by Team } n\} && \text{for } n = \text{A, B} \\
b_1 &= \Pr\{\text{1-pt. conv. attempted by Team B}\} \\
b_2 &= \Pr\{\text{2-pt. conv. attempted by Team B}\} \\
t_n &= \Pr\{\text{TD by Team } n \text{ in 1 possession}\} && \text{for } n = \text{A, B} \\
g_n &= \Pr\{\text{FG by Team } n \text{ in 1 possession}\} && \text{for } n = \text{A, B} \\
z_n &= \Pr\{\text{no score by Team } n \text{ in 1 possession}\} && \text{for } n = \text{A, B} \\
r &= \Pr\{\text{Team A wins in overtime}\}
\end{aligned}$$

- What is the relationship between b_1 and b_2?

$$b_1 + b_2 = 1$$

- What is the relationship between t_n, g_n and z_n?

$$t_n + g_n + z_n = 1$$

- What is the probability that Team B scores 0 after a touchdown?

$$1 - (\underbrace{b_1}_{\text{Team B goes for 1?}} \underbrace{p_{1B}}_{\text{Team B makes 1?}} + \underbrace{b_2}_{\text{Team B goes for 2?}} \underbrace{p_{2B}}_{\text{Team B makes 2?}})$$

3 The stochastic DP

- Stages:

$$
\begin{aligned}
t = 0, 1, \dots, T-1 \quad &\leftrightarrow \quad \text{end of possession } t\\
t = T \quad &\leftrightarrow \quad \text{end of game}
\end{aligned}
$$

  - For our purposes, a possession ends when a team scores (TD or FG), or loses possession without scoring

- States:

$$
\begin{aligned}
(n, k, d) \quad \leftrightarrow \quad & \text{Team } n\text{'s possession just ended} && \text{for } n \in \{A, B\}\\
& \text{Team } n \text{ just scored } k \text{ points} && \text{for } k \in \{0, 3, 6\}\\
& \text{Team A is ahead by } d \text{ points} && \text{for } d \in \{\underline{\dots, -1}, 0, 1, \dots, \}
\end{aligned}
$$

$d < 0 \Rightarrow$ Team B ahead by d points

- Value-to-go function:

$$
\begin{aligned}
f_t(n, k, d) = \text{ maximum probability that Team A wins when in state } (n, k, d) \text{ at the end of possession } t\\
\text{for } n \in \{A, B\}, k \in \{0, 3, 6\}, d \in \{\dots, -1, 0, 1, \dots\}
\end{aligned}
$$

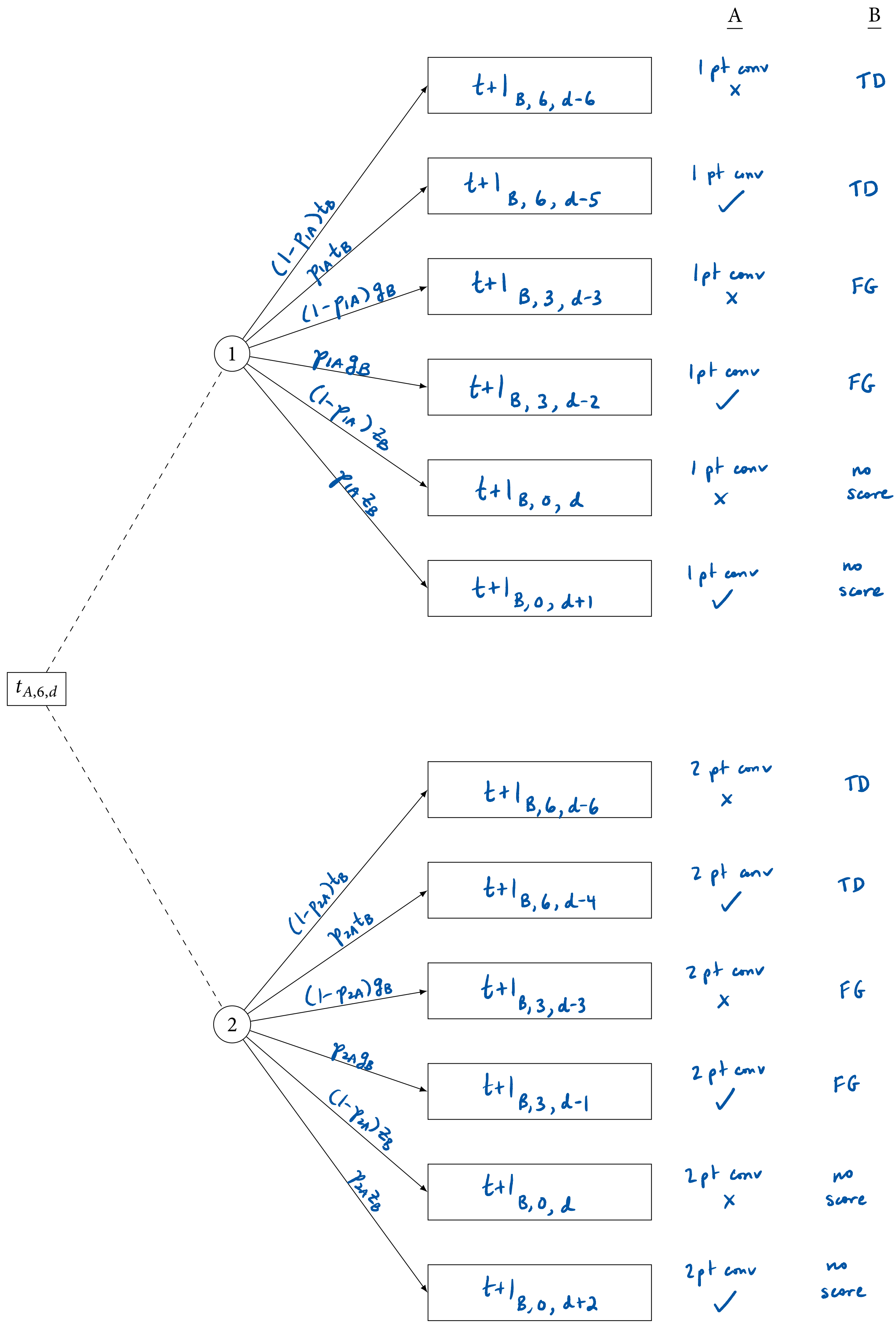
- Allowable decisions x_t at stage t and state (n, k, d):

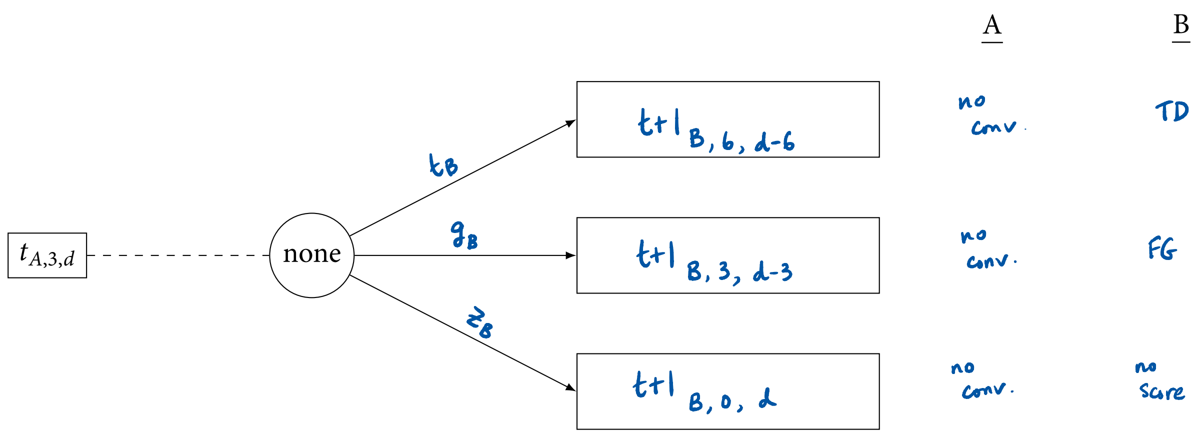
$$
\begin{aligned}
x_t &\in \{1, 2\} && \text{if } n = A \text{ and } k = 6\\
x_t &= \text{none} && \text{if } n = A \text{ and } k \in \{0, 3\}\\
x_t &= \text{none} && \text{if } n = B \text{ and } k \in \{0, 3, 6\}
\end{aligned}
$$

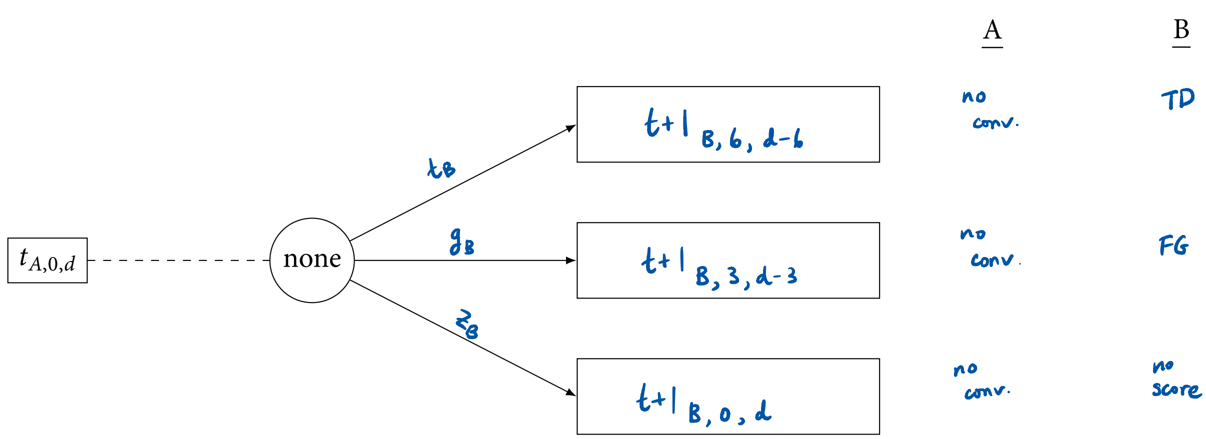
- We need to consider transitions from the following states:

$$
\begin{matrix}
(A, 6, d) & (A, 3, d) & (A, 0, d)\\
(B, 6, d) & (B, 3, d) & (B, 0, d)
\end{matrix}
\qquad \text{for all } d
$$

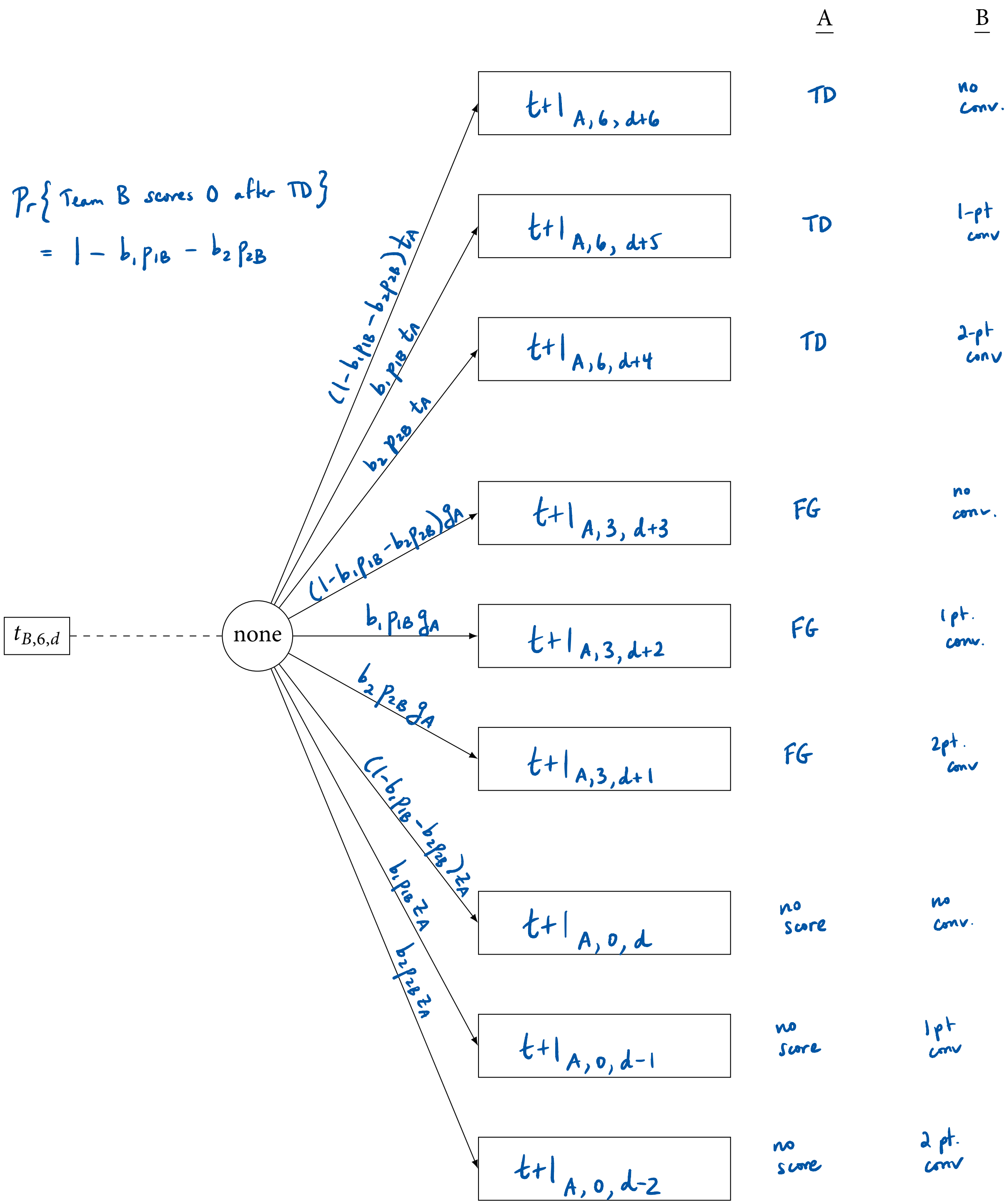
- Since our objective is to maximize the probability of winning, we set all the contributions in stages $t = 0, 1, \dots, T-1$ to 0, just like in the investment problem in Lesson 13

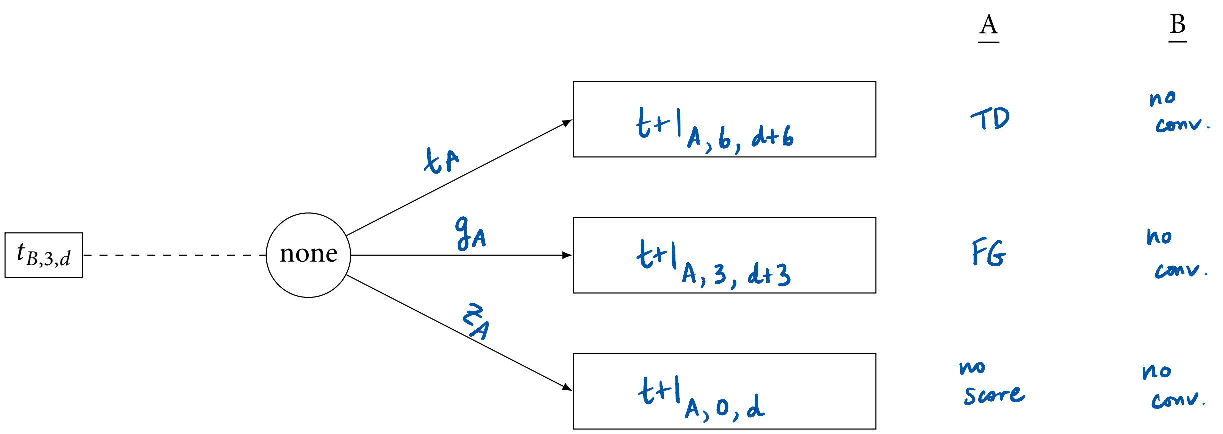

A
B
$t_{A,6,d}$
1
2
$(1-p_{1A})t_B$
$p_{1A}t_B$
$(1-p_{1A})g_B$
$p_{1A}g_B$
$(1-p_{1A})z_B$
$p_{1A}z_B$
$t+1_{B,6,d-6}$
$t+1_{B,6,d-5}$
$t+1_{B,3,d-3}$
$t+1_{B,3,d-2}$
$t+1_{B,0,d}$
$t+1_{B,0,d+1}$
1 pt conv ✗
1 pt conv ✓
TD
FG
no score
$(1-p_{2A})t_B$
$p_{2A}t_B$
$(1-p_{2A})g_B$
$p_{2A}g_B$
$(1-p_{2A})z_B$
$p_{2A}z_B$
$t+1_{B,6,d-6}$
$t+1_{B,6,d-4}$
$t+1_{B,3,d-3}$
$t+1_{B,3,d-1}$
$t+1_{B,0,d}$
$t+1_{B,0,d+2}$
2 pt conv ✗
2 pt conv ✓

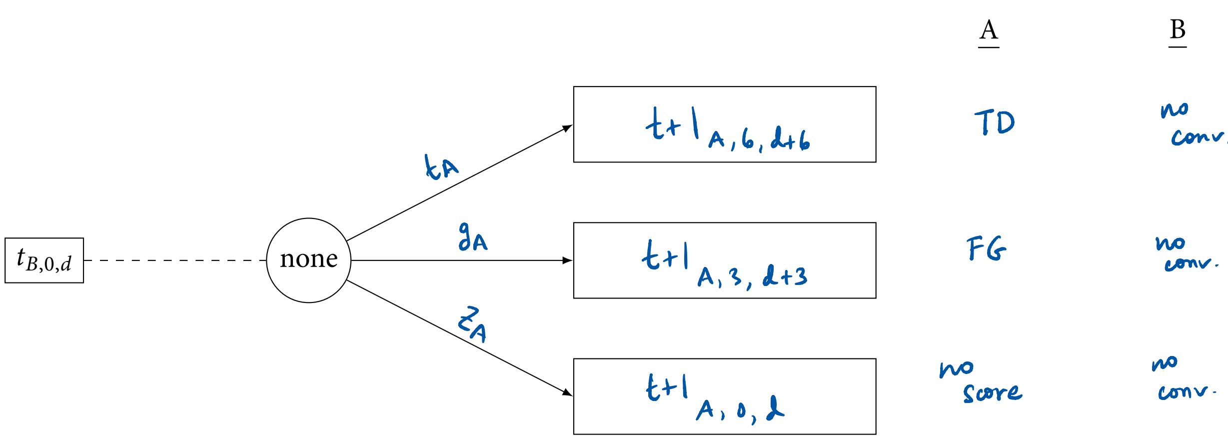

A
B
$t_{A,3,d}$
none
t_B
g_B
z_B
$t+1_{B,6,d-6}$
$t+1_{B,3,d-3}$
$t+1_{B,0,d}$
no conv.
TD
no conv.
FG
no conv.
no score

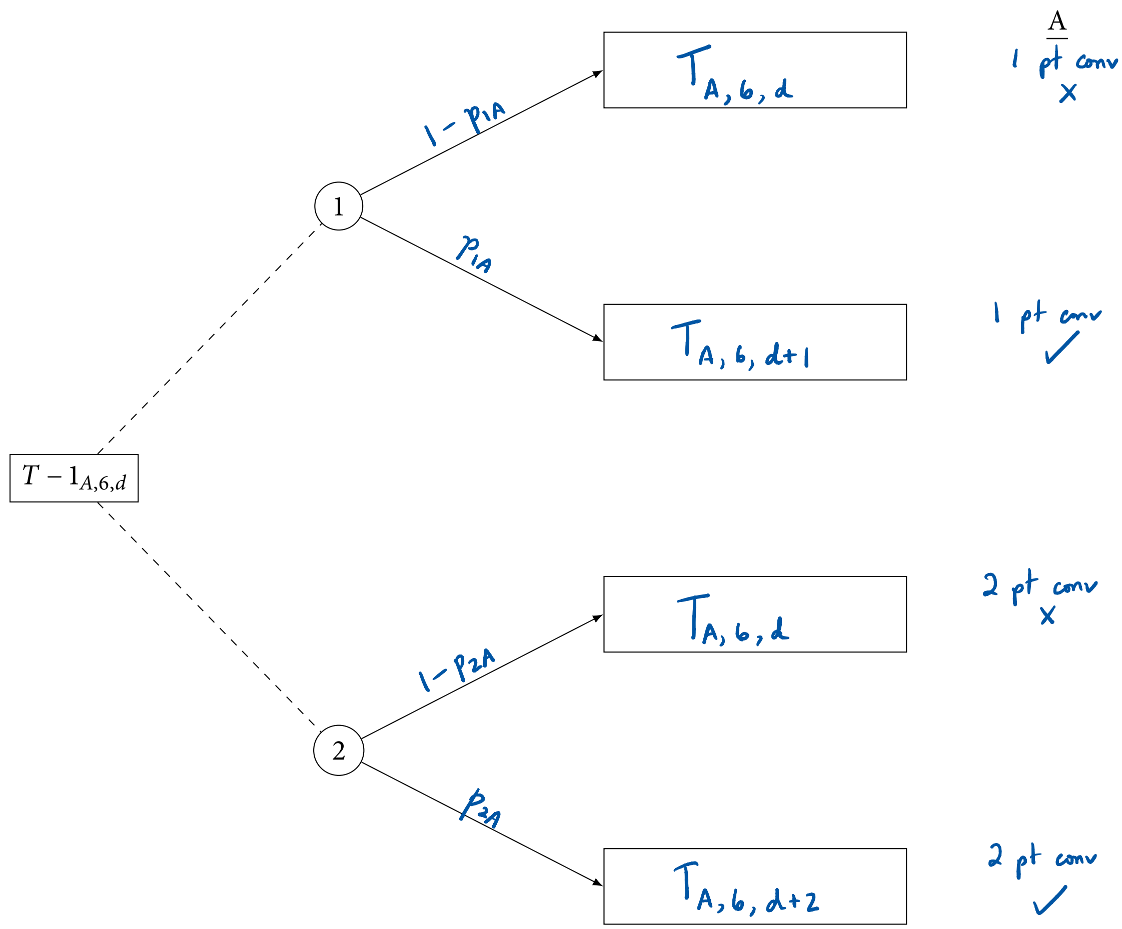

A
B
$t_{A,0,d}$
none
t_B
g_B
z_B
$t+1_{B,6,d-6}$
$t+1_{B,3,d-3}$
$t+1_{B,0,d}$
no conv.
TD
no conv.
FG
no conv.
no score

$\Pr\{\text{Team B scores 0 after TD}\}$
$= 1 - b_1 p_{1B} - b_2 p_{2B}$

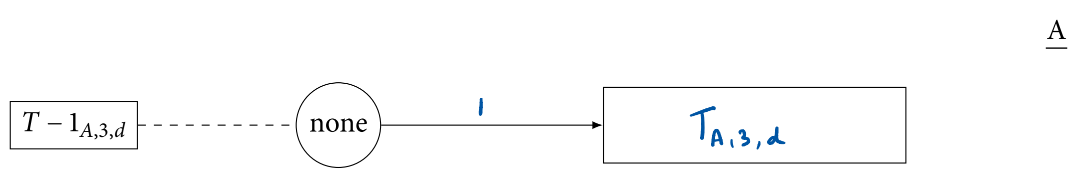



A
B
$t_{B,3,d}$
none
t_A
g_A
z_A
$t+1_{A,6,d+6}$
TD
no conv.
$t+1_{A,3,d+3}$
FG
no conv.
$t+1_{A,0,d}$
no score
no conv.

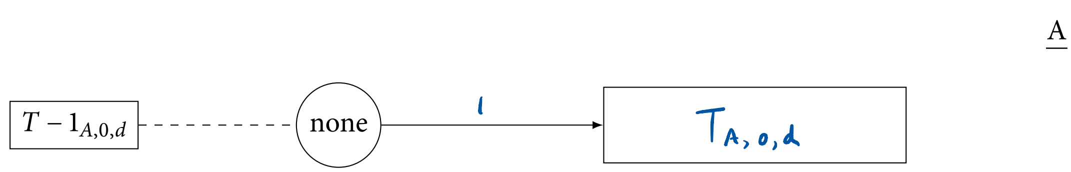

A
B
$t_{B,0,d}$
none
t_A
g_A
z_A
$t+1_{A,6,d+6}$
TD
no conv.
$t+1_{A,3,d+3}$
FG
no conv.
$t+1_{A,0,d}$
no score
no conv.



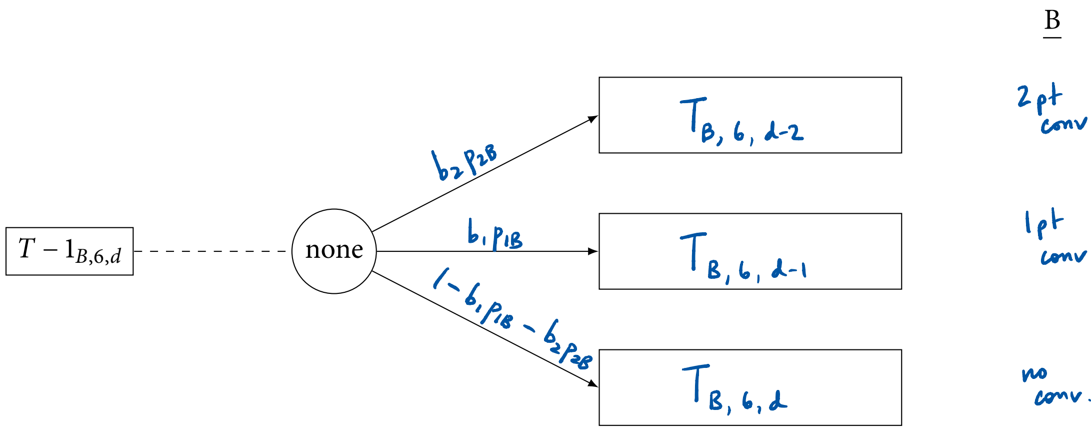
A
$T-1_{A,6,d}$
1
2
$1-p_{1A}$
p_{1A}
$1-p_{2A}$
p_{2A}
$T_{A,6,d}$
$T_{A,6,d+1}$
$T_{A,6,d}$
$T_{A,6,d+2}$
1 pt conv ✗
1 pt conv ✓
2 pt conv ✗
2 pt conv ✓

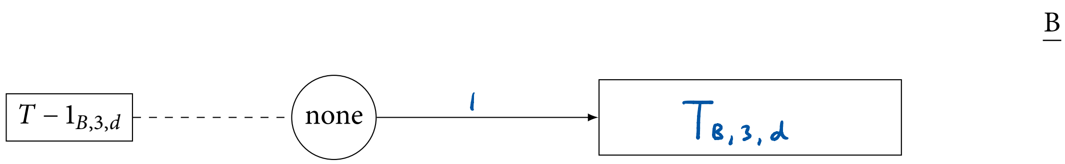


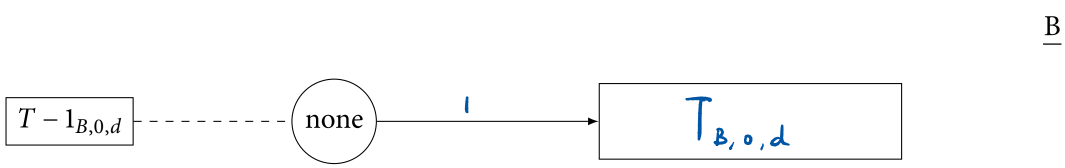
A
$T-1_{A,3,d}$
none
1
$T_{A,3,d}$



A
$T-1_{A,0,d}$
none
1
$T_{A,0,d}$


B
$T-1_{B,6,d}$
none
$b_2 p_{2B}$
$b_1 p_{1B}$
$1 - b_1 p_{1B} - b_2 p_{2B}$
$T_{B,6,d-2}$
$T_{B,6,d-1}$
$T_{B,6,d}$
2pt conv
1pt conv
no conv.


B
$T-1_{B,3,d}$
none
1
$T_{B,3,d}$


B
$T-1_{B,0,d}$
none
1
$T_{B,0,d}$